
Is This Penny Fair? (Spinning Pennies)

Teacher Notes

Common Core State Standards

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

2. *Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Preparation

I used 1959D pennies for this activity. These pennies can be bought for a reasonable price (\$3-\$5 for a roll of 50) at a coin store. Furthermore, I had these prescreened by having students spin them during an activity period (it's surprising to me that they actually enjoyed spinning pennies for long periods of time).

Commentary for Teachers New to Statistics

This activity and the concomitant commentary are intended primarily for non-Statistics teachers who need to implement some Statistics in their classrooms. However, for current Statistics teachers, this activity can serve as a nice introduction to Statistical inference (making decisions from sample data).

The spinning pennies activity is really what statisticians call a significance test (or a hypothesis test) in disguise. The ultimate goal of a significance test is to weigh the evidence against an established belief. This established belief is called the null hypothesis and is denoted H_0 . In this case, the null hypothesis is that in the long run, the proportion of times a penny will land tails up when spun is 0.5 ($H_0: p = 0.5$, where p is the true probability of a tail).

When conducting a test of significance, we have another belief that we suspect might be true. This is the alternative hypothesis and is denoted H_a . In this case, the alternative hypothesis is that the proportion of times the penny will land tails up when spun is greater than 0.5 ($H_a: p > 0.5$, where p is the true probability of a tail).

A test of significance assumes that the null hypothesis is true, and then checks sample data to see if the data are reasonably consistent with what we would expect if that null hypothesis really were true. When the sample data stray too far from what is expected, we have evidence to doubt the null hypothesis. Consider a fair penny that is spun 50 times and which lands tails

up 34 times. Probability theory tells us that the probability of 34 or more tails out of 50 spins about 0.8% of the time. This probability – that we see an outcome as extreme or more extreme than the one observed from our data, assuming the null hypothesis is true – is called a *p*-value. If we see 34 or more tails, *this does not mean we have proven that the penny is unfair*. After all, 34 or more tails does happen by chance about 8 times in 1000. However, if we have to make a decision about that penny with the evidence of only one set of 50 spins, we would reject the null hypothesis and instead choose to believe that the penny is not fair, which is called the conclusion of our test of significance.

Teacher Notes

In the real world of Statistics, certainty is absent. We must make decisions in a world of chance and uncertainty. This is an important point to drive home to your students. It is not possible to be certain about our decision, but we can inform our decision through an analysis of that patterns established by random chance.

The table in question 5 is an important part of engaging student intuition. It is NOT very surprising to see 26 or 28 tails in 50 spins of a fair coin. However, by the time students get to 45 tails out of 50 spins, most students would agree that they would switch their beliefs and declare the coin as unfair. This reasoning process reflects the decreasing probability that these outcomes happen just be chance. In essence, students are using their intuition to draw a conclusion about a hypothetical test of significance.

The histogram provided in this activity is a from a computer simulation of spinning a fair penny. This was provided so that students would not need to be burdened with complex probability formulas. My current understanding of the Common Core State Standards is that students would not be expected to calculate this probability theoretically, but from simulations as in this activity. They would be expected to make decisions based on data and evidence from simulations.

Students will likely need some help through questions 10 and 11. This is the crux of the reasoning process in a test of significance and will require a little bit of careful thought. Keep reminding students to look at the probability (percentage of times) of seeing as many tails as they saw *or more*.

Proposed Solutions

1. Answers will vary based on student intuition.
2. Answers will vary based on student data.
3. 25 tails.
4. No, sometimes there will be more or fewer tails than 25.
5. Answers will vary based on student intuition. By 45 tails, most students should believe the coin is unfair.
6. Answers will vary based on student intuition.
7. Yes, it is possible. No, it is not likely.
8. The penny landed tails up 34 or more times in 7 of the 1000 sets of 50 spins.
9. The probability is 0.007 or 0.7%.
10. I would now believe that it was unfair. Although it could have landed tails up 34 or more times just by chance, that chance is very small.
11. Answers will vary based on student data.